Mathematics

A conversation with Afshah Deen and Emma Turner

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In terms of mathematics, what do you want your Year 6 pupils to know, understand and be able to do when they finish KS2?

By the end of Year 6 we want children to be creative and curious about mathematics. We want the pupils to be able to solve unfamiliar, complex problems. We want them to be resilient when they're solving problems. This means being flexible in their approach, being able to solve problems in lots of different ways and being able to listen to other people's approaches to problem solving and learn from them. If a child in Year 6 is faced with a problem and they're starting to solve it in one way, and then they start to realise, 'Oh, this isn't going to help me to be successful in solving this,' we want them to think, 'Okay, where have I gone wrong? Have I got the right end of the stick? Do I need to go back and reread it? Do I need to solve it in a different way?' We want them to be flexible in their mathematical thinking and have a range of mathematical processes at their disposal to solve problems using the most appropriate methodology. It's important to encourage the metacognitive process of talking through solving problems, so when they come to problems of a slightly different nature to those already encountered, Year 6 pupils can have that critical, questioning, metacognitive rumination about how they would approach new problems, and think about what they know about a certain aspect of mathematics and apply it to something unfamiliar.

Underpinning it all is pupil confidence, and that comes with procedural fluency. Pupils need a sound understanding of the fundamental mathematical processes. Knowledge and understanding at Year 6 are linked to pupils' competences in the core operations – place value; the four operations; and fractions, decimals, percentages, ratio, and proportion (FDPRP) – which, combined, help pupils unlock problem solving.

For example, having a deep understanding of the basics of number is crucial, which means knowing and understanding securely: place value, the four operations, and fractions. And being able to explain what those things are before they can reason logically about mathematical concepts, which leads eventually to solving problems in a range of different and appropriate ways. Ultimately, when they're able to integrate procedural fluency, conceptual understanding and problem solving, and can solve unfamiliar, complex problems efficiently and creatively, *then* they are decent mathematicians.

We must be careful, however, because it's a big misconception to think that it's *all* about *procedure*. Mathematics is about understanding the connections between all the ways that number works, how number relates to things like time, to data, to statistics, to shape, to angle. It's about understanding the interconnectedness within the discipline rather than just learning a sequence of procedures, and that's where people go wrong because they teach the procedure, but not the concepts or the connections within number that give children the deeper understanding of how the number system works. You need the procedures and you need an understanding of the connectedness.

When we're teaching in a connected way in primary, it's making sure that we represent the numbers – the linear number system – in multiple ways for children to understand deeply how the number system is constructed. For example, for a great deal of early mathematics you're asking children to work in the abstract. You're asking children to imagine what these numbers look like, how big they are, how many of them there are, how six is different from seven, how four looks different from 10, and notionally how far apart three and 10 might be if you laid them out. We're asking children from the very start to work in the abstract. To connect things, especially in relation to early number, requires both the pictorial representations and concrete representations of number to match the abstract concepts.

Now, the problem in primary is that we do this well in Early Years and in KS1, and then after that we expect children to have totally internalised the number system and have these mental representations that they can draw on, and then to work in the abstract from the beginning of KS2. What we want to do, if we're going to teach in a connectionist way, is to build up this small bank of reliable mental models supported by things like Cuisenaire rods or Numicon shapes or Rekenrek bead strings or whatever it is that you want to be using, so that by the time they get to Year 6 the pupils have their own concrete representations of the number system in their heads to use when problem solving. So, you take that number system to explore place value in a careful way. For example, bead strings are organised red and white normally, but they're organised in fives, so you can see those boundary numbers. If you've got the little bead strings that go up to 20, they're organised in five, so they're based on that idea of subitising, so that you can see when you've got seven beads, you've got five of one colour and two of another. You can see that representation there as well.

When we construct a curriculum for mathematics in primary, what we need to look at are the representations that we are using with children when we are deliberately representing mathematical concepts at specific points in their learning. Then we need to consider how we're asking children to represent their thinking back to us in their own words. And, as they move through different concepts, we want children to be able to articulate to us what they are thinking constantly. We want them to do that through them explaining those internalised models and internalised representations. What you don't want to do in a primary curriculum is have a very concrete and pictorial representation-heavy curriculum in KS1, and then move into KS2 and say, 'Oh well, we do it all in our heads now,' because what you're constantly doing is exploring further and further within the number system. For example, in Years 3 and 4 when you start to explore decimals, you can't do it unless the children have that pictorial and concrete representation of decimals in their minds, otherwise you're asking them to imagine a concept that they've never necessarily encountered.

Initially there can be an overreliance on working in the abstract, and an overestimation of how much children can do and have genuinely internalised in terms of understanding the number system. Those countries that have truly successful mathematics curricula are the ones that spend an enormous amount of time ensuring that children have got secure representations, so they spend a great deal of time practically exploring mathematical concepts through Variation Theory. They do this with practical equipment, so that pupils can secure those understandings and those connections. We have an assumption in KS2 that the children have internalised the number system and truly understand all the connections between it. However, whenever you encounter a new part of the number system, or a new concept or a new approach to something, it needs to be represented in both concrete and pictorial ways before pupils can access the abstract. This links with Cognitive Load Theory which is based on the idea that our working memory – the part of our mind that processes what we are currently doing – can only deal with a limited amount of information at one time.[1]

1 https://impact.chartered.college/article/shibli-cognitive-load-theory-classroom/

If you are asking children to imagine something, that's a huge amount of working memory you're taking up. What you want to do is to say, 'Look, here is the representation, let's explore what that looks like, how we can manoeuvre it around, how we can play with it, how it looks like something we've done before.' And, if you're going to teach the connections well in mathematics, it is important to limit the number of things that you show the children in terms of equipment at the initial point of interaction with the new concept. You wouldn't use double-sided counters on Monday, Rekenrek bead strings on a Tuesday, Numicon shapes on a Wednesday and then cut out laminated kites on a Thursday. Children can't make those connections with such a varied range of physical representations of number. Once children have secured the initial understanding you can expand the variety of representations. It's those deliberate curriculum decisions about equipment as much as it is about delivery that matter. It is tempting to rush through the curriculum without emphasising, for example, how crucial place value is, and how it links to things they are doing, or will be doing, in Year 6. Pupils need the chance to unpick numbers and understand the relationship between those numbers. How many ways can you make six? How many ways can you make five? For example, with partitioning in Year 2 and Year 3, if you ask a child to partition 35, they might just say 30 and five. In fact, there are so many other ways of partitioning 35. It's important for pupils to be flexible, and to appreciate the making and breaking of numbers, which is at the heart of partitioning numbers. And that principle applies to all aspects of the fundamentals of teaching and learning mathematics.

Furthermore, there needs to be a focus on mental mathematics. When we worked in the mathematics consultancy team for the national strategies, we analysed thousands of SATs papers. What we found was that those children who were mentally fluent with mathematics on the mental mathematics paper were much more likely to do well on the written test. It was those children who had a secure understanding of number and rapid access to internalised mathematical facts and representations, who had that mental fluency, and that fluency spilled over into their written methods. By contrast, what we focus on, slavishly sometimes, is evidencing things in books and getting stuff written down. There is a rush to standard written methods when the time needs to be taken to develop secure mental calculation. And this relies on well-internalised pictures of the number system alongside an ability to represent their thinking in non-standard methods. The rush to pencil and paper, and to evidence that we're doing 'hard mathematics and standard written methods' is detrimental to pupils' understanding.

Teaching the underlying structure of the number system

Counting underpins everything that we do in mathematics. When we say counting, we think about simple recitation of number names but there are so many parts to counting. It's the number names and knowing those number names in a stable order, so it goes one, two, three, four, five – it doesn't go one, three, four, six, seven, eight, three.

First, you've got to learn the songs, like *Five Little Ducks and One, Two, Three, Four, Five, Once I Caught a Fish Alive*, which aids the pupils to know the names of the numbers in order. Then there is one-to-one correspondence. This means one number name goes with one object, and that's hard, and a hugely important stage. This is because children will point and count, but they need to get the point, the word and the object together because what you end up with is a child pointing and saying 'five, six, seven, eight, nine, 10, 11,' and actually there are only three objects there. They are just saying number names and pointing at things, but getting the number name, the object you're pointing at and the right number of objects to tally is a huge developmental step for children.

So, you know the number names, you know them in order and that order is stable and never changes. That's the first stage. The next stage is that one number name matches one object, be that finger counting or whatever. Children may have problems with that; they may duplicate the number and say 'one, two, two, three, four' and overcount. They might not know when to stop, so if you've got blocks on a table that they're counting, they'll just carry on forever more, and they don't know which ones they've counted. They may not know that you must count all the objects; once they've counted just a few they will say, 'I'm done now – that's how many there are.' One way we can get round that is to encourage children when they count, to move those objects physically, put them in a line and begin to internalise that representation of the linear number system.

The next part is about knowing the difference between the cardinal and ordinal concepts of counting. So, if you show pupils three fingers and then ask them, 'How many fingers am I showing?' the answer's not, 'One, two, three,' the answer's three, but lots of children will automatically say the answer, 'One, two, three.' The answer is 'three' and that requires careful teaching. Then they need to understand that if you say, 'Give me three cubes from your line of cubes,' they need to give you *three* cubes, but often they'll just give you the third one. They also may, at this point, think that each individual cube just has a name, 'The number two cube, the number four cube…' They don't understand that the number four cube is the fourth in that group of four.

We then start to explore this idea of subitising; being able to count without having to count. It's based on how you can count up to five objects straight away. Because you have five digits on your hand, you can just naturally recognise five. Subitising is important; when we come on to things like partitioning numbers in different ways, being able to recognise five and a *bit* is crucial. If seven is five and two more, you can see that straight away – it's one of the connections with how seven is built up.

So, once we can do subitising then we move on to abstraction within counting. We count things that we can't move, things that are imagined, things that are at a distance. We count things of different sizes or count things that are different distances away from each other. So, if you've got a ball on the carpet, a ball near the door and a ball in the corridor, that's still three; even though they're far apart, there are still three things there. If you've got three cubes and five cubes and you push the five together so they're really close and then spread the three out, you ask, 'Which one do you think's got the most in it?' Finally, there's the idea of conservation of number, that it doesn't matter if you mix things all around again, there's still the same number there. So, you have a red cup, a blue cup and a yellow cup in that order; when you switch the order around you've still got those three cups. Conservation of number is key. Children right up to Years 1 and 2 can have real gaps in understanding conservation of number and will only believe that it's a seven if it's presented in a specific way, in a specific order, in a specific representation – they can't make that conceptual leap.

So, those are the steps in counting and teachers will often say, 'Well, I've done counting,' or claim the pupils can count. But the other things that they need to be able to do within counting is to count things in lines and count things that are positioned randomly as well. They also need to be able to recognise within the linear number system the relative position of numbers for counting. 'Can you give me an example of two numbers with a really big gap between them on this number line?' They might say two and 17. 'So could you give me two numbers with a really small gap between them?' They might say seven and nine are really close together. They can't do that without having a good understanding of counting and number. Counting backwards is the absolute crux of subtraction. People like to count forwards and don't like to count backwards but we do that scrolling through the number system, we go forwards and backwards. Then counting on from any number, not just from zero or from one – count on from seven, count on from 22, count on from 0.6. The reason I've thrown decimals in there is because whenever you explore a new part of the number system, you need to renew the pupils' understanding of all these parts of the number system. With counting and number recognition, they also need to recognise the most significant digit is on the left, that it's like reading, that we go left to right. We also need to be able to order numbers; not just, six, seven, eight, but 23, 17 and six. That's based on your understanding of counting.

Conceptually you need to explore other aspects of counting… 'Can you count things that you can't touch, that you're not allowed to get your hands on? What would be your strategy for counting the things you can't actually physically move?' Then you begin to link the idea of counting to addition, counting forwards for addition and counting backwards for subtraction. You must count on different equipment. How do you count on a number grid? How do you count on a number track, or on a number line? How do you count on a partly numbered line where there are only the fives or only the 10s shown?

With early mathematics, there is the issue of language. I don't mean mathematical language, but everyday language: 'before', 'after', 'above', 'below', 'beneath', 'less than', 'fewer than', 'more than'… If you're going to be looking at a 100 square and the children don't understand the concept of 'next to' or 'behind' or 'beneath', they can't navigate that 100 square to make sense of the mathematics. Consequently, we must develop the vocabulary that facilitates counting. Furthermore, it is important to understand number as a noun rather than an adjective. So, when I say three, I mean the number three, not three balloons or three teddies or three kites or whatever it is. 'When I say three, what do we mean by three? What does three look like? What is three not like? How is three different from four? How is three different from zero?' Moreover, if subtraction is always talked about as something that has been taken away, or something is less, then the term 'difference' doesn't make sense because that hasn't been taken away, so the language needs to be clear. This also applies when we are using bar models: if children do not understand the relationship and the language that is used as 'wholes' and 'parts', it is hugely problematic.

There is also the idea of being able to count things that are different sizes, so that if you've got three beach balls that's not more than three golf balls just because the beach balls are bigger. And then, for example, having a beach ball, a golf ball and a pea, have you still got three? Well, yes, there are still three there, even though those three items are different. These are concepts many children can really struggle with. If you show them three little teddies and three cars, it's getting them to understand there are still three, even though one is three teddies and one is three cars; so, understanding what we mean by counting, and those stages in counting, is enormous. Then you've got the whole business of understanding the actual representation of the number, how much that means; it's a bit like phonics in that what you actually write bears no resemblance to the meaning of the number just as the way you write the grapheme 'f' bears no resemblance to the actual sound it represents. If you are writing a number seven, you've only got two lines, but actually that's the number seven and that corresponds to seven objects. They could be seven big objects, seven small objects, a mix of different objects, arranged in a different order. Later, pupils need to learn how to write a tally of seven, so that they understand all the representations of seven.

The other thing about counting and early number is getting children to recognise numbers in their environment, and a lot of the numbers, unfortunately, in children's environments don't relate to anything. Like the number 37 bus is not the 37th bus that has come along that morning, it's just the number 37 route. Or the number 8 on the back of a football shirt doesn't mean that that person wearing it is eight years old, it's just a position on a field. This is the nominal aspect of number – numbers just used as labels. Children need to understand that numbers represent both actual amounts and can be used as an abstract concept as well. It is about getting them to recognise the numerals, getting them to understand one-to-one correspondence and getting them to understand that we use numbers for different things.

Once we've mastered counting, we can begin to explore some of the aspects and properties of number. Partitioning numbers in different ways is important. If we're looking at seven, how can we partition seven into five and two, six and one, seven and zero and understand that additive identity of zero as well, that zero exists but it doesn't change the number if you add anything to it? So, we partition numbers, not just into 10s and ones but into all the possible combinations for each number, initially up to five, then to 10, then to 20. Getting those secure is crucial because those number facts, number bonds and the partitioning, underpin everything in calculation. When you get to formal, compact written addition, when you add and carry, if you don't know those quick number facts and which ones will bridge over 10, then that becomes a complicated process. Compact addition is underpinned by all the work that goes on in KS1 and Early Years, in terms of understanding counting, understanding partitioning, understanding the properties of numbers. When we talk about fluency and automaticity in early mathematics, that is it. If we're adding eight and three, they should know straight away that the quickest way to do that is to do eight add two add one, rather than eight, nine, 10, 11 because they know that three is made up of two and one. All of that comes from this early work on counting and early number understanding. In KS1 and Early Years we must not rush to the procedural – we should be staying at this conceptual level and really developing this deep understanding of the number system.

During your Foundation Stage and Year 1 you need to be so focused on getting that internalised understanding of what the number system looks like, how it functions. 'Do we understand the proximity of numbers to each other? Have we grasped the conservation of number? Do we understand how to break numbers up into their constituent parts, not only in 10s and ones? For example, 15 could be represented by 10 and five, and it could be nine and six.' There are so many other ways of breaking the numbers up. Understanding how we organise the number system in terms of those boundary numbers is key too – where are the 10s? 'What's special about the 10s? What's different about the 10s compared to other numbers? How close are we to 100? If you put 46 down, what's 46 near to? What is it miles away from?' It's really about understanding, and beginning to internalise that linear number system, and being able to present it and talk about it in all sorts of different ways.

As with anything, you want children to make connections with number. What we need to connect is this idea of language, visuals, concrete examples and symbols. You've got the symbolic nature of mathematics with the numerals; you've got the language that you use; you've got the pictures that you want children to see and to represent; and then that concrete experience. So, everything we explore needs to have those four elements; that's what both facilitates and constitutes connectionist teaching. Some of my go-to pieces of equipment include a number track, a number line – the difference between a track and a line being that a track has no intermediate value, it's like bums on seats in a theatre. Number one sits in the number one space, number two sits in the second space and nobody sits in between the seats. A number line you can start to stretch, which is interesting; clearly you can explore a number line as single intervals and then you can show them that between nought and one, there are other numbers and you can show there are numbers in between each part of the number line.

Bead strings are just a joy. Initially we use a 20-bead string that is based on subitising. It would be normally red and white in fives, and so you begin to see the relationship in subitising, the bridging. Then the 100-bead string is a great representation of 100. You can start to use it again when you go back to looking at decimals because one complete bead string is one, so rather than having 100 individual things it could be the tenths you consider which are neatly represented there, as are the hundredths. Then using double-sided counters is so important, where the yellow side is positive and the red side is negative. They're great for subtraction too because you literally flip the counters over. Rather than taking them away, you can say, 'I've changed them from being red to yellow – what has that done to the problem now?'

You can use those in what we call tens-frames, which are basically a little five-by-two grid with five on the top and five on the bottom. You can start to fill the frames with the counters, and you can see how many more you would need to make ten. Base 10 equipment and Dienes blocks are still my go-to; you have the tiny little Dienes cubes for the ones, which then become the sticks of ten, which then become the flats of 100, which then become the cubes of 1000. It's so easy to represent what numbers mean with those. When you're doing things like exchanging and you take nine units, add one more and that becomes a stick, you can physically see the exchange. The other thing I like about it is the children can draw those easily. So, if they're starting to annotate their own thinking, if they want to do 32 add 23, they can draw three big sticks and two little dots and then another two sticks and another three dots.

They can show their thinking with those manipulatives and their writing as well. Arrow cards are great, the little arrow cards that sit over each other. So, you've got 123 and you can explode it and then you see the 100 and the 20 and the 3 and then you can collapse it back together again, to show place value. I can't advocate practical counting equipment enough, and having cubes, whether that's Unifix or Multilink, in every single classroom. Always encourage children to explain their thinking to you with the equipment. It is a crucially important visual representation of what's going on in their head. If they're not using the equipment to show you what they're thinking, their understanding is completely hidden from you. If they're in literacy and they're writing a story, you can hear their words, they're on the page; if they're calculating something and the answer is eight, you've got absolutely no idea how they got that eight. By them manipulating that equipment, they are articulating their thinking processes and you can begin to interrogate them. You can ask, 'What would happen if I did this?' – early conjecture, early hypothesis, early reasoning. They can also begin to explore and explain to other people. So, equipment is so important, for every context that you teach, be that with just counting in early number or when you're starting to explore decimals or fractions; having that equipment there is so important for children.

The first of the four operations: addition

When we come to look at the four operations, there is an order that builds on that initial grasp of the number system. For example, if we take addition to begin with, it is commutative, which means that you can move numbers around and still arrive at the same answer. You can do seven add three, three add seven, and you end up with the same answer. The interesting learning point is that our representations need to match that. If we're showing seven add three, we need the seven objects under the seven things and the three objects under the three things. I've seen so many times where they've been the wrong way round and it doesn't match. Making the models and the images match is so important.

There are two main parts to addition, there's augmentation and aggregation. Augmentation is where you are counting on from a number, so if you're doing seven add three you might say, 'The plant was seven centimetres long and it grew another three centimetres, how tall is the plant now?' Then aggregation is where you're combining two sets or two groups. Children need experience (but not necessarily the language) of both, because if they're going to be those brilliant problem solvers, they need to be able to visualise what's happening in that problem. Is this an augmentation problem or is this an aggregation problem? Augmentation draws on that skill of counting and the aggregation will draw on that skill of understanding numbers and how they're put together. They need to understand that before we get to bridging. Then for aggregation we can begin to use basic addition and equals symbols: this circle with five objects in and this circle with two objects in added together makes seven objects in this final bigger circle. With augmentation we can use things like number tracks and number lines, where we say, 'Right, we're on the seven and we're going to go along three more.' That is the fundamental thinking behind addition.

Initially, in terms of progression, you would want children to explore calculations where they're counting on but there's no bridging; for example, calculations like six add two or eight add one, where they haven't got to go over the 10. From counting on with no bridging comes counting on where you're using number bonds, where you will get to the next boundary number, so six add four, five add five, eight add two. Then you'll do numbers that bridge, so you might do eight add six, and you'd bridge.

Then you would teach compensation; for example, you would do 23 add nine. You'd do 23 add 10 and take the one off. You would compensate for what you'd added. That goes back to that idea of constant counting, so counting in decades is another part of counting that you really need to do. Counting in decades, not just 10, 20, 30, 40, 50, but counting in decades, 23, 33, 43, 53, because that all feeds into when you come to do things like compensation. One thing that we miss is near doubles for addition. So, if I said to you, calculate 22 add 23. Instead of 20, add 20, add two, add three, you'd say 22 add 22, add one, using and applying the knowledge of near doubles. Then finally, partitioning the second number only. If you wanted to do 32 add 21 you would break that down to 32 add 20, add one equals 53. You would not do 30 add 20, two add one; there's a very good reason for that because you can't apply that double partitioning for subtraction in the same way because it depends on the size of the units number, the second digit (e.g. 45 − 26. Many children doing a double partition then mistakenly switch the order of the 5 and the 6 in an attempt to replicate what works within addition and end up with the wrong answer by doing 40 − 20 = 20. 6 – 5 = 1 so the answer is 21 rather than 19). So, when teaching children addition, to partition the second number only draws on their experience of counting and it's a transferable method that they can use in subtraction, whereas double partitioning, partitioning both numbers, doesn't always work in subtraction depending on the size of the numbers.

I always say that when you teach all these strategies, an equal amount of time needs to be spent practising the picking of the strategy as much as the doing of the strategy. So sometimes I would give children 10 calculations and I would say, 'Right, there's a range of addition calculations, I only want you to solve the compensation ones,' or 'I only want you to solve the ones with near doubles.' They would have to be making those critical decisions about what method they would use. I would then go on and say, 'Okay, well there are three calculations that you would use with compensation, I now want you to write down three examples of questions with near doubles for *me* to solve.' So, they would be thinking about not only how to do the strategy – because they'd have to work out the answers so that they knew if I was right or not – but also, they're making critical decisions about the structure of the number system and how to calculate well. So, the practising of the selection of the method is as important as the teaching of the method itself. They are effective problem solvers, not merely careful rule followers.

The second of the four operations: subtraction

Subtraction is two principles again, taking away and difference. Firstly, they need the experience of having a number and taking some objects away; however, the worst thing to do is to physically take them away.

What you want to do when you do take away is to move them. You've still then got the concept of the whole number sentence. So, using 10 objects, if you did 10 subtract seven, you'd move the seven to the side, you would still see the three, and you would then still see that seven and three ultimately made the 10. So, removing them, even though that's the process, isn't helpful for developing that conceptual understanding.

The next bit is to find the difference. Despite the common feeling that it's hard to teach difference, it isn't if you begin with a very visual, immediate example. Show two people of different heights and say, 'Jane is this tall, Sasha is this tall, what's the difference between them? My teddy is this high, your teddy is this high; what's the difference between them?' So that again goes back to counting. If you can count on, you can find the difference. It's pretty easy as well to do difference because it's counting forwards. Counting backwards is where children get in a mess. Counting forwards to find the difference is straightforward. The progression in mental calculation for subtraction is very similar to addition. So, count back first of all, so eight subtract two or nine subtract three, then count back where you have to bridge over the 10, which again goes back to your understanding of partitioning. And then there's compensation, but this time you'll take off 10 and add one back on because you took off too much, so you need to compensate for what you did. Then counting on where there's a small difference. So, for example, 21 subtract 18, they can see that small difference and calculate that easily. Then there is partitioning the second number only, like addition. So, consider 63 subtract 28: we'd do 63 subtract 20, subtract eight, because you can't do 60 subtract 20 and then three subtract eight. Double partitioning doesn't work for subtraction, so partition the second number only. These are all mental methods because what we want children to do is to look at a calculation and think, 'What's the best way of solving that?' What children do is default to the last strategy they were taught rather than looking at the numbers and thinking, 'What's the most efficient way to do this?'

The third of the four operations: multiplication

Multiplication is commutative and associative. So, you can do it in any order, like its cousin addition, and you can regroup numbers using brackets and you will get the same answer. So, you can break it down even further. So, if you're going to do 12 multiplied by five, which is 60, you could do 10 multiplied by five and two multiplied by five and add those together and you'll still get 60. This is useful for teaching multiplication tables: if you're going to teach your sevens and the children go, 'Oh, the sevens are really hard, I can't do sevens,' well, you can do fives and you can do twos and because multiplication is associative, do the fives and the twos and add them together and there's your answer. It's built on that understanding of how the number system and the structure works. Using the commutative and associative properties of multiplication to teach the multiplication tables is key.

The other thing about multiplication is that it is repeated addition. If you've got four multiplied by three, that's four and four and four, it's four three times. What people get in a mess with is the operator/operand relationship. They think that four multiplied by three is four threes rather than four, three times, but in the calculation the operand and the operator are in a very specific order and if we get them the wrong way round, we're teaching children the wrong way round for multiplication. The order of operations is important. Ultimately it doesn't matter in terms of the answer because it's commutative, but, actually, we need to be mindful of matching our calculations with the actual operator/operand order.

For multiplication, you start by counting in steps of different size – so you count in fives, count in threes, count in eights, go back to that idea of counting but this time we're counting in different steps, but using the same modelling. You teach that repeated addition about the fact that – and we will model it as actually repeated addition – three counters and three counters and three counters and three counters, is three four times. Begin with three, then add three, add three, and add three. Or we could say it's three four times, but we must model that. We also then start to use the array (an arrangement of numbers in rows and columns, or just the plain dots, on a grid). Now that again is beautiful because you can go back to what you understand about partitioning, back to the idea of the associative aspect of multiplication and you can start to show with the array how you can multiply by seven easily. You do your rows of sevens and then you put the line down between the five and the two and you say, 'Look, here's our five times table, here's our two times table, we can derive this,' but too often the array and grid method is only used for partitioning calculations into 10s and units. When, for instance, you're multiplying 17 by six using the partitioning method, you'll see 10 and seven multiplied by six; you can break it down even further if the children can't access it, by doing 10 and five and two multiplied by six, on the array. Understanding the array feeds into grid multiplication, which is a beautiful bridge in terms of children's understanding of what multiplication is. Then you can start to do short multiplication, but we need to spend an awful long time with the models and the representations of early multiplication as repeated addition and use the array so they can understand what we mean by it.

Then linked to that is the idea of doubling. If you want to multiply by four, double it, double it again or multiply by eight, double, double, double. So being able to double and being able to halve is important in terms of mental calculation. On the old mental mathematics paper, pupils were more likely to get an old national curriculum level 4 or level 5 if they scored highly on the mental mathematics test rather than on the written mathematics test and it's because those children who can calculate mentally have a robust understanding of the number system. Those children who can do procedural calculations and perform well on a written paper but haven't necessarily got that mental fluency are the ones where it all comes unstuck when they start exploring other parts of the number system.

The last of the four operations: division

When you are understanding multiplication to begin with, you must grasp that concept of repeated addition. You're talking about a group repeated again and again, and that's an important concept to understand because that's one of the concepts that underpins division. The reason a lot of children can't get to grips with division is that it's incredibly heavily reliant on the other three operations. You have to draw on your knowledge of addition, subtraction and multiplication to be able to understand division securely, which is why developing those early building blocks – building fluent and automatic understanding – and being able to access those number facts across the other three operations, is important. If you don't understand division, then fractions are just an absolute chaotic circus. That's why children get in such a mess with fractions, and why lots of primary school teachers find teaching it really challenging. It's not the actual content of fractions – which is straightforward – it's because fractions are so inter-reliant on the other four operations.

In division, we want to develop that concept of what's happening within division, rather than rushing to a formal method. When anybody thinks of division, a lot of the time they think of short box division, sometimes called the 'bus stop' because it looks like an old bus stop. In terms of understanding division to start with, you're thinking about two different concepts. The concept of grouping and the concept of sharing. Now, both of those are division, both of those are valid, but one becomes very inefficient very quickly. Sharing is a natural concept for children to understand. They have to share in class, they have to share at home, they have to share their sweets. Even if they're playing alone, they will be sharing toys, sharing soldiers inside and outside a fort, perhaps? Sharing is a natural splitting concept that they understand. You can work out the division calculation by sharing – one for me, one for you, one for you, one for you, one for you. However, as the dividend becomes bigger and the divisor becomes bigger, that becomes massively inefficient, so you can only actually really use sharing practically when you're dealing with exceedingly small numbers. And it relies on one-to-one correspondence. You're not actually doing any calculation. You're just picking up things, putting them in hoops or circles or whatever it is, or teddies on a picnic table or whatever it is, and then counting how many are in each group.

You would build on the children's natural experience of sharing, by taking a group or a set and splitting it in some way. As you begin to explore larger numbers, bigger dividends with bigger divisors, you will start to think, 'Well actually, if I want to do a 1000 shared between 20, I'm going to be here all day. I need a different method to do this.' What you can start to look at, and you can do this with smaller numbers to start with, is to think about grouping. If you had 12 divided by three, it's asking yourself how many groups of three are there in 12. What you have is your 12 in a line and you start to grab those groups of three, then you count how many groups of three you have. So instead of doing a one-to-one correspondence, you're starting to think of it as an actual group. Now, it's not forced at all because if you're thinking about children's toys and play, you might say, 'How many cars can I make from 12 wheels if all the cars need four wheels?' You can start to relate it to things that they're making, things that they're doing. You'll say, 'Right, instead of going one, two, three, four, let's take four and put those on, and another four and put those on…' so you can do it really practically to start with, but they need to get this idea that division is both sharing, but also it's about removing a group or creating a group from a bigger group that you start with, or a bigger quantity that you start with.

Now, you can then begin to relate it to what they've understood in multiplication because there's that connection between multiplication and division. If you've got 12 divided by four, how many groups of four are there in 12? Well, there's three. You can check that by saying four, three times is 12. So, if you already understand your multiplication facts and you can derive that, seeing the division is really easy. By using the same equipment that you would've used for multiplication, for example the array, you can start to see the groups on the array when you've laid that out in that grid, and you can start to circle the little groups in the array and you can actually see it. The children haven't had to think of anything new or get to grips with any new equipment. They're seeing the same image, just interrogated in a different way, and you've made that solid connection between what they already knew and what they're learning now.

Grouping is being able to start with that bigger number and split it into groups. Children can start to record that on a number line, so they can start to record their division by doing hops on the number line either forwards or backwards. I would say if you're going to record it on a number line and you're going backwards on a number line, put your jumps underneath so that you don't lose track of what you're doing, so you can see that we're talking about jumps of, or groups of, rather than individual numbers. You'll start to ask children everyday questions, for example, 'If a tent can sleep five people and there are 20 people going on the trip, how many tents are we going to need, because they're going to have to be in groups of five?' You'll get them to explore this idea of putting things into groups which throws up the concept of remainders. Things don't naturally fall into perfect groups every time. There may well be always one left over or two left over, so you can just talk about remainders; for example, if you had an answer of three remainder one, you'd say, 'Well, there are three whole groups and one left over out of the next group of three. I've got one of the next group of three left over.' It's not just a random one. You want to get children to develop that language of, 'I'm putting things in groups of three, and this is one left over that would've been in the next group of three'. That's important for understanding a third; if you understand division, you'll understand fractions, so you start doing grouping. Then you can start to calculate using your knowledge of grouping and your knowledge of associating multiplication facts. If you're going to start grouping, mentally, if you're going to go through a mental progression of it, first of all it would just be those individual groups of the divisors, so you would do 12 divided by four, or you'd do 16 divided by four or 20 divided by five. You would actually get them to say that. 'How many groups of five are in 20?' You want them to be internalising that concept of, 'This is about groups within a larger set.' You start by doing that.

The next step is to use a chunk of 10 of the divisor. For example, you might do 36 divided by three, so rather than saying, 'Well, we could go three, and three and three and three,' you say, 'We could do 10 lots of three which uses up thirty straight away, and then two more little groups of three is six, so actually it's 12 groups of three.' We've got 10 add two which makes 12 groups, but if they haven't understood their multiplication and place value, they will never make the conceptual link that you can do 30, a group of 10 of those groups of three, first. Then you can extend that to be groups of a hundred – chunks of a hundred of the divisor. You might do 336 divided by three, so they're getting this idea that you can divide like that. Then you'll go back and you would say, 'Okay, how about if we did 83 divided by six?' The children are like, 'Oh, hang on a minute, that six isn't the same as the 80, the six... What do I do?' Again, you model it all and you show them what's happening. The temptation here is to start teaching the box method of division. There's plenty of time for developing procedural fluency. When you're looking at 83 divided by six, you'd do 60 which would leave you with 23 and then you'd think right, how many more groups of six can I get out of that 23? Then you expand that out and you think, 'Okay, well how about if I do something like 430 divided by six, how do I do that? What do I know about my multiplication tables? Well, I know about 42 and a six, so I can do a really big calculation if I apply what I know about place value and multiplication. I know six sevens are 42, so I can do a big jump of 420 and then do that last little bit that's left,' drawing hugely on what you understand about the number system, what you know about all the interrelations between all of the other operations, and your mental agility, and you still don't need to write that down. If you jump to that procedural point straightaway, you are denying the children the opportunity to understand what's happening, to build that picture in their minds of what's happening in the calculation. Even in Years 2 and 3 they should still be building this big picture of the number system.

Fractions

When we talk about modelling any calculations for children, there's a difference between a model and an image when we're teaching mathematics. The modelling is the annotation on the board of how you solve the problem, so if you're going to do compact addition you write your 43 at the top and your 26 at the bottom and you draw two lines, you add those two up first, those two up next, you might have some carrying, blah, blah, blah, that's modelling. That's the 'what you do'. That's what you do to solve this problem. That's the process that you go through. That needs to only happen when the children have got the conceptual understanding to underpin that because they can't remember what they need to do to solve the problem and that is where their confidence drains away. That's why they get it wrong because it's easily forgotten. You know what you need to do but you've forgotten the process, so when we're modelling, we need to be mindful about whether we are modelling a process or are we using imagery? By mathematical imagery, I mean the 'why this works, why we set it out in this way'. If you're starting to lay out a calculation, have you got it modelled with the numbers but then also by the side of it what it actually looks like in terms of the physical equipment? 'What is happening to the numbers as you model that, where do you move the equipment to, why does that become a stick of 10 and move over and that becomes your carrying on that side? Are we modelling and using images at the same time?' We want children to use imagery constantly in their explanations so that we can interrogate it. We need to ensure that our imagery on the board can be interrogated by the children who can ask, 'I don't understand why those three counters go over there. I don't understand where that one came from.' Likewise, when they're calculating with equipment, we need to be able to say, 'What would happen if I did this, or does this still work if I put two more there?'

The imagery that we use we need to be able to maintain all the way through. That is especially important for fractions!

There are four main representations of fractions: fractions as part of a whole thing, like a cake; fractions as part of a set, like a group of people; how we use fractions to model division and then how fractions are linked to ratio. The first thing that children get wrong with fractions is linked to their day-to-day experience, because adults in their lives will talk about the bigger half: 'Who wants the bigger half of this piece of cake?' It's not a bigger half, it's either a half or it's not, and that's a concept called equipartitioning. Children need to understand fractions as the idea of equipartitioning, equal groups, which again is handy if they've understood grouping and division because those groups were equal, so we talk about equipartitioning. When we first introduce fractions, we need to have a circle with a line drawn exactly down the middle, so you have two equal halves, and then a circle with a line just drawn towards the edge. You'll say, 'Okay, they've both got two pieces in each of those, so they're both halves, aren't they?' You want the children to say, 'No they're not, they're not.' 'Well, why not?' 'Well, they're not the same,' so you then start to talk about what we mean by 'the same', and how we mean 'equal', they are 'equal to each other'. Those two parts are equal parts, and so we teach children the concept of equipartitioning.

The fraction notation throws children as it turns everything they understand about number on its head. If you've got the notation for a half and then the notation for a third, in a child's world, three is more than two, so a third must be bigger than a half. This is where you need to go back to the physical/pictorial and say, 'Okay, I've got your favourite chocolate bar here; would you like me to split it in half and give you a piece like that, or would you like me to split it into thirds, into three equal pieces and give you one of those?' And they reply, 'No, I want the half, I don't want the third.' You must teach the concept that says the more bits you split the whole into, the smaller each piece is; so, you've got more bits of the whole and that's where the bigger number comes from, but actually each individual bit is smaller. The bigger the number the smaller the bit. They need lots of practical explanation to grasp that concept. And we would put 1/1 to illustrate the whole one.

We then need to explore non-unit fractions, so looking at things like four fifths or three quarters and explaining how we're not talking about one of those pieces, we're talking about more than one of those pieces. That's where we can start to explore that link between the false rule about 'the bigger the denominator, the smaller the fraction' because you can say, 'Well what about 50 over 100 compared to a third? Which one do you want then? If we cut it into 100 equal pieces, that's quite a lot of additional tiny pieces, but if I gave you 50 of them, would you want that or would you want a third?'

One key thing that we don't do, and we wonder why children get confused, is to keep the same size image or the same shape image when they're making those links. So instead of talking about thirds and they're a triangle, or quarters and that's a square, you keep the same size, same shape thing that you're cutting up, so they can directly compare between the images. You'll start with unit fractions, then simple fractions like three quarters, but then you'll start to explore the representation of it. This is the notion we call quantification, where if you think about a quarter, one over four, children will see that as one thing over another four things (and make five things) because they think it's a direct relationship between the one and four when actually what you're talking about is one over another three things, because all together, that group is four things. It is important to explore the notation around that thoroughly so that they've got that real concept of quantification, and when we are writing this down, they understand what it actually means. We're talking about one out of these four things, but actually it's one and then another three of those to go together.

The poor relation of fractions – and it shouldn't be – is this idea of reconstituting; so, if you've got a third, you've not got the other two thirds, and getting children to understand that one third goes with two thirds to make three, which is the whole. A bit like you would teach a fact family like seven and three is 10, three and seven is 10, 10 subtract seven is three, 10 subtract three is seven. It's the same with fractions; one third and two thirds is a whole one and getting them to understand that relationship between the bits that are there and the bits that are not there. The other thing that they get confused by is the language; for example, if we're doing fifths. You've got a square and you've cut it into strips that are equal fifths, and you'll say, 'Okay, show me a fifth.' Very often they will show you the fifth one, not necessarily a fifth, and they won't understand necessarily that all those strips are fifths. There's number one fifth, number two fifth, number three fifth, number four fifth and there's a fifth fifth.

Then you'll start to look at finding fractions of a set, or fractions of a quantity, and this is where it relies heavily upon multiplication and division. It's where you have to really start to draw it. This goes back to the reason we say it's not that sharing is wrong, it just becomes inefficient. It's linked to multiplication; the commutativity of multiplication is important here, because if we're going to find one third of six, we're not saying six shared, we're actually saying six shared between three, but if we were going to work it out mentally, we're going to use six divided by three. How many groups of three are there in six? So, it becomes this flow between sharing and grouping, multiplication and division, and if those things haven't been firmly secured to start with, this is where pupils' mathematics comes undone. The children don't know where the numbers are coming from; so, you can start to model finding fractions by sharing and grouping. If I were going to calculate one third of six, I would draw a circle and I would divide it into thirds, so I'd say, right, I know what a third is. A third is one equal part of the three equal parts of this. Then I can get six counters and I can put one in there, one in there, one in there, one in there, one in there, one in there, and I can start to see that there would be two counters in each one of those thirds, so I'm making that link between the practical pictorial shape exploration and then that conceptual leap into doing it with number. I'm bridging the gap between them. They end up with two counters in each one of their thirds, so a third of six must be two because there are two in each one. Then they suddenly realise, 'Well, I can just do six divided by three which is two. I know that because I know my division facts, I know my multiplication facts, and I can start to apply what I know to this problem.' Initially with unit fractions you would only find one third of something, and then you would say, 'Well, what about if I want two of those groups of three, what would I do then?' They would say, 'You just add the two together or multiply them together.' And that's your link to multiplication, but unless multiplication is strong and understood as repeated addition, they're not necessarily going to get that concept. That's why fractions become so tricky because it leans on everything else that's come before.

When I divide by the denominator, I'm actually putting it into those equal groups, I'm creating the equal groups and then when I'm multiplying by the numerator, I am adding the appropriate number of groups together. At the point of any assessment of children, I would always go back to the concrete and the pictorial and say, 'Show me how this works, show me how I would find this out using these tubes or this bead string. Describe the process to me.' I'm not really interested in the right answer. I'm interested in how they process it.

Once they've mastered being able to find a unit fraction and/or fractions which have numerators greater than one, you can start to look at equivalents. Equivalents can get confusing because people will say that a half *is the same as* two quarters. But it's not the same as two quarters, it's *equivalent* to two quarters. It's not the *same*. When we're talking about equivalents, we need say 'is equivalent to' not 'is the same as', and that's something that we teach in calculations. The equals sign does not mean here comes the answer. It means 'is equivalent to' and that's why when we're teaching any calculations at all, any of the four operations, we need to present children with calculations where the equals sign is in different positions. Otherwise, they just think, 'Here comes the answer' and that's unhelpful for later in their mathematical development when they're doing lots of things about algebra and balancing equations.

Any new kind of concept that's being taught needs to be explored conceptually in lots of different ways before rushing through to the calculation element. If we don't spend longer on teaching and developing concepts through Variation Theory, exploring them in lots of different ways – and a fraction is a good example of that – then there are lots of misconceptions that children are likely to pick up. Fractions are not very difficult if children have been exposed to lots of different ways of looking at fractions. Usually we show children the whole, 'Here is a whole, find me the part. Find me one sixth. Find me one eighth.' Now, why not show the part? Begin with the part and say, 'Make me the whole.' It's important to show fractions in many different ways and not just say, 'Here is your pizza!'

Delay the use of pen and paper

I want children to meet a calculation and ask themselves, 'Do I really need to write anything down? Can I do this in my head?' You don't want children to have a default of just picking up a pen every single time. You want them to look at that number and think, 'What's the most efficient way of doing this?' A great game that I do with the children is to 'Beat the Calculator'. I will write a calculation on the board and they're working in pairs. One of them has a calculator and one of them has their head.

The person with the calculator must press every single button to get the answer; the person who can do it mentally doesn't have to. Who's fastest? You can do that in a trio where somebody's got a pen and paper and must do the written calculations, one's got to do it mentally, and one's got the calculator. Who's fastest? They love 'Beat the Calculator', but it's showing that it's much swifter to do things mentally a lot of the time if you've got all those skills.

All the processes like bridging, partitioning, near doubles and compensation, comprise a toolkit which I want children to have in their head, to be able to look at a calculation and think, 'Which tool in my mathematical toolkit am I going to use for this one?' The last resort is the pen and paper. Now, there comes a point where you could work a relatively complex calculation out in your head, but it becomes inefficient and that's the point at which you resort to pen and paper. We feel this pressure to rush to pen and paper in mathematics, but it's different from literacy. There is a push to write because writing is a skill and a means of communication. In mathematics, yes, you can communicate your thinking through writing but to be good at mathematics, you want to be able to be a good mathematician in your head as well. Whenever we're teaching anything, any of the four operations, any of the concepts, you want those four points of that diamond, symbols, language, pictures, and concrete experience.

So only when the children have really internalised their concrete experience, they've developed their own pictures of what the number system looks like in their head, they've got all the associated language to be able to talk about that and frame their thinking, that's the point where you introduce the symbols and get them to do the appropriate notation and what have you.

I'm sure it's the national curriculum where it says mathematics provides fresh avenues to explore for the curious mind. It talks about an inherent beauty of mathematics and there is both simplicity and complexity. There is awe and wonder when you're looking at mathematics in nature, when you're looking at mathematics in parks. It's gorgeous to look at. When you teach so that the children have a profound understanding of the number system and the four operations, you're opening this magical world where everything fits together, and if you explore it, you can really start to make deep connections and they love that. Children love pattern making, they love pattern spotting. I don't mean about putting counters in the shape of a circle or whatever, but actually looking at creating number sequences with patterns. When you start colouring multiples on a number line or a hundred square and seeing the actual layout of the pattern that it produces, they're absolutely blown away by stuff like that. What we focus on, for some reason, is shutting that down and getting children to put a pen in their hand and solve page after page of calculations. I understand the need for fluency in a written calculation, but that should be the absolute end game really.

It's the same with problem solving in mathematics. You want to teach children lots of different ways of looking at the number system and attacking it, so that when they're presented with a problem again, they can choose which strategy to use and choose which method to use. If you want children to do a page of written calculations, or a page of solving worded problems, give them a mixed set and say, 'I only want you to solve the ones that have got addition in them,' or 'I only want you to solve the ones where you think the answer would be greater than 100.' You're constantly promoting that decision making alongside the fluency because, otherwise, children are just passively writing down calculations.

Final reflections

It's place value, four operations and a sound grasp of early fractions that open the door to the whole rest of the mathematics curriculum. It's that practising the selection of the most appropriate method based on a really sound understanding of the number system and of place value, supported by deliberate connections between well-chosen pieces of equipment. You should use the same pieces of equipment or familiar equipment for multiple areas of the mathematics curriculum, so that children are making those connections. As a subject leader, it's making those very deliberate choices in our school at the point of new conceptual understanding: 'This is what we use and we've made this deliberate choice to use this, to represent it.'

It's not about providing children with 25 different methods to solve something. It's saying as a school, we really value these five. We think these are efficient. We think these work. We can represent them well with the equipment that we've got, and it will give children that fluency that they need to be able to solve these calculations. You need to teach them a bank of methods they can pick from and then practise picking from that bank. As a subject leader for mathematics, deciding how your school will present number to children is an absolutely core part of your work, and then all the interventions that you do and the support that you provide should be linked to the way that you as a school are incrementally developing mental calculation strategies, alongside written calculation strategies, and how you present the number system to children. That should be systematic, it should be strategic, it should be incremental. It should have absolute clarity about it.

As a subject lead in my school, I knew if I walked into Year 2 they would be able to calculate with these sorts of numbers in these sorts of ways in these four operations. Then I could go into the Year 4 classroom and I would know these children are going to be calculating in this way with these numbers at that point. We trained the staff and the support staff to say, 'When we're talking about calculation, this is how we do it here.'

The only other thing is ensuring that the children are calculating within the part of the number system that they're fluent in. It is important that you calculate and problem-solve with each section of the number system, so that children gain fluency, and you explore the next stage of the number system to develop that understanding of it, but you don't calculate within it. For example, if you've only explored numbers up to five, you will do your problem solving within that, but you may be exploring some of the numbers up to 10, but you won't be problem solving up to 10. Gradually, as you become more fluent with the next part of the number system, say fractions, that's when you'll start to do the problem solving in fractions. It's a little bit like when you first learn to drive and you pass your driving test, you just want to drive to places that you know rather than go out on the motorway. If you're having to have some of your cognitive resources outsourced to look at the problem-solving aspects of what you're doing, then doing that in a completely unfamiliar territory of the number system as well is just horrible. So, calculate and investigate with numbers that you're fluent with and gradually, in the background, you start to build up that wider understanding of the number system which you can then explore through problem solving.

Background: mathematics

From 3000 BCE, arithmetic, algebra and geometry were developed in Mesopotamia for commerce, to record time and to work out calendars. The Pythagoreans were the first to refer to mathematics and were mostly concerned with deductive reasoning. Chinese mathematics developed a place value system and the concept of negative numbers. The Indo-Arabic numeral system was invented between the 1st and 4th centuries in India and adopted by Arabic mathematics in the 9th century. Algebra was invented by the influential Persian polymath Al-Khwarizmi.

It is worth quoting the purpose of the subject from the national curriculum programme of study:

> 'Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.
>
> 'The national curriculum for mathematics aims to ensure that all pupils:
>
>> 'Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately; reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language; can solve problems by applying their mathematics to a variety of routine and non-routine

> problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.
>
> 'Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.'[2]

Once the importance statements have been revisited, it is helpful for subject leaders and co-ordinators to discuss and agree with colleagues the reason why their subject, in this case maths, is important for the pupils in their school. One way of doing this is to draw on a quote, in this case from Albert Einstein: 'Pure mathematics is, in its way, the poetry of logical ideas.' This kind of prompt allows us to formulate our way of stating the importance of the subject. We might agree or disagree with such a statement and in doing so come to a form of words which expresses our view of the importance of this subject, in this school. This moves us away from the territory of 'we teach this subject because of the SATs or GCSEs'. While the external tests and exams are important, they are not the totality of the subject.

Professional communities

Subject associations are important because at the heart of their work is curriculum thinking, development and resources. There are a number of subject associations for mathematics and the National Centre for Excellence in the Teaching of Mathematics[3] lists these on its website. Any member of staff with responsibility for a subject should be a

2 Department for Education. (2013) *National curriculum in England: mathematics programmes of study*. Available at: www.bit.ly/3mrWjoT (Accessed: 9 March 2022).

3 www.ncetm.org.uk

member of the relevant subject association, and this should be paid for by the school.

Twitter subject communities are important for the development of subject knowledge, because it is here that there are lively debates about what to teach, how to teach and the kinds of resources that are helpful. For maths, it is worth following the NCETM on Twitter[4] and the hashtag #mathscpdchat.

Links

Mathematical Association – www.m-a.org.uk
National Centre for Excellence in the Teaching of Mathematics – www.ncetm.org.uk
La Salle Education – www.completemaths.com

4 www.twitter.com/ncetm